

**DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE TOLD TO DO SO**

T.B.C. : B-GSE-P-TUB

Test Booklet Series

Serial No.

0003825

IES/ISS EXAM, 2016  
**TEST BOOKLET**  
STATISTICS-II



**Time Allowed : Two Hours**

**Maximum Marks : 200**

**INSTRUCTIONS**

1. IMMEDIATELY AFTER THE COMMENCEMENT OF THE EXAMINATION, YOU SHOULD CHECK THAT THIS TEST BOOKLET **DOES NOT** HAVE ANY UNPRINTED OR TORN OR MISSING PAGES OR ITEMS, ETC. IF SO, GET IT REPLACED BY A COMPLETE TEST BOOKLET.
2. Please note that it is the candidate's responsibility to encode and fill in the Roll Number and Test Booklet Series A, B, C or D carefully and without any omission or discrepancy at the appropriate places in the OMR Answer Sheet. Any omission/discrepancy will render the Answer Sheet liable for rejection.
3. You have to enter your Roll Number on the Test Booklet in the Box provided alongside. **DO NOT** write *anything else* on the Test Booklet.
4. This Test Booklet contains 80 items (questions). Each item comprises four responses (answers). You will select the response which you want to mark on the Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose **ONLY ONE** response for each item.
5. You have to mark all your responses **ONLY** on the separate Answer Sheet provided. See directions in the Answer Sheet.
6. All items carry equal marks.
7. Before you proceed to mark in the Answer Sheet the response to various items in the Test Booklet, you have to fill in some particulars in the Answer Sheet as per instructions sent to you with your Admission Certificate.
8. After you have completed filling in all your responses on the Answer Sheet and the examination has concluded, you should hand over to the Invigilator **only the Answer Sheet**. You are permitted to take away with you the Test Booklet.
9. Sheets for rough work are appended in the Test Booklet at the end.
10. **Penalty for wrong answers :**  
**THERE WILL BE PENALTY FOR WRONG ANSWERS MARKED BY A CANDIDATE IN THE OBJECTIVE TYPE QUESTION PAPERS.**
  - (i) There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, **one-third** of the marks assigned to that question will be deducted as penalty.
  - (ii) If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that question.
  - (iii) If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.

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1. For what value of  $\alpha$ , the BLUE of  $\beta_1$  and  $\beta_2$  are uncorrelated for the model  
 $E(y_1) = 2\beta_1 + \beta_2, E(y_2) = \beta_1 - \beta_2, E(y_3) = \beta_1 + \alpha\beta_2$  ?  
 (a) -1  
 (b) 0  
 (c) 1  
 (d) 2
2. Let  $y_i \sim N(0,1)$  where  $i = 1, 2, 3, \dots, n$  be  $n$  independent standard normal variates, and  $y' Ay$  be a quadratic form where  $A$  is an idempotent matrix, then what will be the distribution of  $y' Ay$  ?  
 (a) Normal distribution  
 (b) Chi-square distribution  
 (c) Bivariate Normal distribution  
 (d) Rectangular distribution
3. A certain task can be accomplished in a factory by four different workers on five different types of machines. A sample study in context of a two-way design without repeated value is being made with two fold objectives of examining whether the four workers differ with respect to mean productivity and whether it is same for five machines. The researcher involved in this study reports gathered data as under :  
 Sum of squares for variance between machines = 35.2  
 Sum of squares for variance between workers = 53.8  
 Sum of squares for total variance = 174.2  
 The F-statistic between machines is equal to :  
 (a) 0.95  
 (b) 1.24  
 (c) 1.48  
 (d) 1.75
4. The following data pertain to a test involving analysis of variance :  
 Estimate of population variance based on variance among the sample means = 18.50  
 Estimate of the population variance based on the variance within the samples = 12.50  
 The F-ratio for the sample is equal to :  
 (a) 0.68  
 (b) 1.48  
 (c) 6.00  
 (d) 31.00
5. In the Gauss Markov linear model, let  $\hat{y}$  denote the vector of fitted values and  $\hat{\epsilon}$  denote the vector of residuals. Consider the following statements :  
 1. The components of  $\hat{y}$  are pairwise uncorrelated.  
 2. The components of  $\hat{\epsilon}$  are pairwise uncorrelated.  
 Which of the above statements is/are correct ?  
 (a) 1 only  
 (b) 2 only  
 (c) Both 1 and 2  
 (d) Neither 1 nor 2
6. Suppose  $b_1, b_2, b_3, \dots, b_n$  are independent  $N(0, \sigma^2)$  random variables and  $\epsilon_{ij}$  are independent  $N(0, \tau^2)$  random variables for  $i = 1, 2, 3, \dots, k$  and  $j = 1, 2, 3, \dots, n$ . Suppose we observe only  $X_{ij} = b_i + \epsilon_{ij}$  for  $i = 1, 2, 3, \dots, k$  and  $j = 1, 2, 3, \dots, n$ . Then which of the following assertions are true ?  
 1.  $\text{Var}(X_{ij}) = \sigma^2 + \tau^2$  for all  $i, j$   
 2.  $\text{Cov}(X_{ij}, X_{i'j'}) = 0$  for all  $i, j$  except when  $i = i'$  and  $j = j'$   
 3.  $\frac{(X_{ij} - X_{i'j'})^2}{2}$  is an unbiased estimator of  $\sigma^2$  for  $j \neq j'$   
 Select the correct answer using the code given below :  
 (a) 1 and 2 only  
 (b) 1 and 3 only  
 (c) 2 and 3 only  
 (d) 1, 2 and 3

7. Which of the following statements are correct about generalized inverse of a matrix  $A$ ?

1. A generalized inverse of a matrix is defined as  $A\bar{A}A = A$ .
2. A generalized inverse is not unique.
3. A matrix to have generalized inverse must be non-singular.
4. A matrix to have generalized inverse may be either square matrix or rectangular matrix.

Select the correct answer using the code given below :

- (a) 1, 2 and 3
- (b) 1, 2 and 4
- (c) 2, 3 and 4
- (d) 1, 3 and 4

8. Which of the following statements are associated with residuals of regression model?

1. The sum of the residuals in any regression model that contains an intercept is always zero.
2. The sum of the observed values  $y_i$  is not equal to sum of the fitted values  $\hat{y}_i$ .
3. The sum of the residuals weighted by the corresponding regressor variable is zero.
4. The sum of the residuals weighted by the corresponding fitted values is zero.

Select the correct answer using the code given below :

- (a) 1, 2 and 3
- (b) 1, 3 and 4
- (c) 1, 2 and 4
- (d) 2, 3 and 4

9. For a simple linear regression model  $y = \beta_0 + \beta_1 x + \epsilon$ , consider the following statements where  $\hat{\beta}_0$  and  $\hat{\beta}_1$  are estimates of  $\beta_0$  and  $\beta_1$ .

1.  $\text{Cov}(y, \hat{\beta}_1) \neq 0$
2.  $\text{Cov}(\hat{\beta}_0, \hat{\beta}_1) = -\bar{x}V(\hat{\beta}_1)$
3.  $E(y) = \beta_0 + \beta_1 x$
4.  $\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$

Which of the above are correct?

- (a) 1, 2 and 3
- (b) 1, 3 and 4
- (c) 2, 3 and 4
- (d) 1, 2 and 4

10. Which of the following statements are correct in order to obtain the hypothesis  $H_0 : \beta_1 = 0$  for the model  $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$ ?

1.  $y_i$  is  $N(\beta_0 + \beta_1 x_i, \sigma^2)$

$$2. \hat{\beta}_1 \text{ is } N\left(\beta_1, \frac{\sigma^2}{\sum_{i=1}^n (x_i - \bar{x})^2}\right)$$

$$3. \frac{(n-2)\hat{\sigma}^2}{\sigma^2} \text{ is } \chi_{n-2}^2$$

4.  $\hat{\beta}_1$  and  $\hat{\sigma}^2$  are not independent

Select the correct answer using the code given below :

- (a) 1, 2 and 3
- (b) 1 and 2 only
- (c) 1, 2 and 4
- (d) 3 and 4



**For the next three (03) items that follow :**

A company used three different methods to train its employees. The number of units of output produced by different employees trained by the three training methods is given below :

Method A : 50 45 55 44

Method B : 64 48 52 56 44

Method C : 46 42 48 45 57 42

It has to be tested whether the three different methods are satisfactory at 5% level of significance ( Given  $F_{2, 12}$  at 0.05 = 3.89 as the critical value)

11. Which of the following is/are the null and alternate hypothesis for testing whether the mean number of units produced per hour is different for the three different training methods ?

1.  $H_0 : \mu_A = \mu_B = \mu_C$
2.  $H_0 : \mu_A \neq \mu_B \neq \mu_C$
3.  $H_1 : \mu_A, \mu_B$  and  $\mu_C$  are not all equal
4.  $H_1 : \mu_A = \mu_B = \mu_C$

Select the correct answer using the code given below :

- (a) 1 and 3
- (b) 1 and 4
- (c) 1 and 2
- (d) 2 and 3

12. The estimate of the population variance on the basis of the variance among the sample means for the above methods is :

- (a) 45.25
- (b) 48.36
- (c) 52.58
- (d) 55.65

13. The estimate of the population variance within the samples for the different methods is :

- (a) 25.67
- (b) 31.07
- (c) 38.646
- (d) 39.097

14. In the Gauss-Markov linear model, what is the possible number of distinct solutions of the normal equations ?

- (a) Either 0 or 1 or infinite
- (b) 1 only
- (c) Infinite
- (d) Either 1 or infinite

15. If in a linear model  $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$  for  $i = 1, 2, 3, \dots, n$ ,  $E(\epsilon_i) = 0$ ,  $E(\epsilon_i, \epsilon_j) = 0$  for all  $i \neq j$ , then which of the following statements are correct for the method of estimation by least squares ?

1. The least square estimates of the parameters  $\beta_0$  and  $\beta_1$  are determined by minimising the sum of squares of residuals.
2. The observations  $y_i$  are pairwise uncorrelated.
3. The observations  $y_i$  are jointly independent and each  $y_i$  is a normal random variable.

Select the correct answer using the code given below :

- (a) 1 and 2 only
- (b) 2 and 3 only
- (c) 1 and 3 only
- (d) 1, 2 and 3

16. If in a linear model  $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$  for  $i = 1, 2, 3, \dots, n$  where  $\varepsilon_i$  are normal and independently distributed random variables with mean 0 and variance  $\sigma^2$ , the maximum likelihood estimators of  $\beta_0$ ,  $\beta_1$  and  $\sigma^2$  are denoted by  $b_0$ ,  $b_1$  and  $\hat{\sigma}^2$  respectively, then which of the following are correct for these estimators ?

1. The estimators are jointly complete sufficient statistics.
2.  $(b_0, b_1)$  is independent of  $\hat{\sigma}^2$ .
3.  $\frac{(n-2)\hat{\sigma}^2}{\sigma^2}$  is a Chi-square random variable with  $(n-2)$  degrees of freedom.

Select the correct answer using the code given below :

- (a) 1 and 2 only
  - (b) 2 and 3 only
  - (c) 1 and 3 only
  - (d) 1, 2 and 3
17. What is R in linear hypothesis  $H_0: R\beta = r$  with two restrictions  $\alpha_0 = 2\alpha_1$  and  $3\alpha_1 = 4\alpha_3$  in three variable linear model ?

- (a)  $\begin{bmatrix} 1 & -2 & 0 & 0 \\ 0 & 3 & 0 & -4 \end{bmatrix}$
- (b)  $\begin{bmatrix} 0 & -2 & 0 & 0 \\ 1 & 3 & 0 & -4 \end{bmatrix}$
- (c)  $\begin{bmatrix} 3 & 0 & 1 & 0 \\ 0 & -2 & -4 & 0 \end{bmatrix}$
- (d)  $\begin{bmatrix} 1 & 0 & -2 & 0 \\ 3 & 0 & -4 & 0 \end{bmatrix}$

18. Mean square error criteria are used for selection of which one of the following ?

- (a) Explanatory variables
- (b) Model
- (c) Estimates
- (d) Response variables

19. In which one of the following assumptions do the general linear model and generalized linear model differ ?

- (a) Linearity
- (b) Homoscedasticity
- (c) Unbiasedness
- (d) Nonmulticollinearity

20. Consider the following statements associated with  $R^2$  and  $\bar{R}^2$  criteria :

1.  $R^2$  increases with the inclusion of relevant explanatory variable only.
2.  $R^2$  increases whether the included explanatory variable is relevant or irrelevant.
3.  $\bar{R}^2$  decreases if the included explanatory variable is irrelevant.

Which of the above statements are correct ?

- (a) 1 and 2 only
- (b) 1 and 3 only
- (c) 2 and 3 only
- (d) 1, 2 and 3

21. Let  $X_1, X_2, X_3, \dots, X_n$  be i.i.d.  $P(\lambda)$  variables. Which of the following statements are correct ?

1.  $T = \sum_{i=1}^n X_i$  is a complete sufficient

statistic for  $\lambda$

2.  $\bar{X} = \frac{T}{n}$  is the unique UMVUE for  $\lambda$

3.  $E(X_i | T)$  is the unique UMVUE for  $\lambda$   
Select the correct answer using the code given below :

- (a) 1 and 2 only
- (b) 2 and 3 only
- (c) 1 and 3 only
- (d) 1, 2 and 3

22. Consider the following statements :

Statement-I : Where the parent distribution is discrete, the non-randomised tests may not exactly attain a given level of significance  $\alpha$ .

Statement-II :  $\alpha$  usually has a low value like 0.01 or 0.05. Even inclusion of any single discrete point with positive probability in the critical region often makes the level greater than the prescribed value.

Which one of the following is correct in respect of the above two statements ?

- (a) Both Statement-I and Statement-II are true and Statement-II is the correct explanation for Statement-I
- (b) Both Statement-I and Statement-II are true but Statement-II is not the correct explanation for Statement-I
- (c) Statement-I is true but Statement-II is false
- (d) Statement-I is false but Statement-II is true

23. Consider the following statements :

Statement-I : Sequential Probability Ratio Test (SPRT) requires, on an average, smaller number of observations than a non-sequential test with identical error controlling properties.

Statement-II : SPRT can be carried out graphically.

Which one of the following is correct in respect of the above two statements ?

- (a) Both Statement-I and Statement-II are true and Statement-II is the correct explanation for Statement-I
- (b) Both Statement-I and Statement-II are true but Statement-II is not the correct explanation for Statement-I
- (c) Statement-I is true but Statement-II is false
- (d) Statement-I is false but Statement-II is true

24. Let  $X_1, X_2, X_3, \dots, X_n$  be independent random variables distributed uniformly on the interval  $(0, \theta)$ . If  $n$  is odd, which of the following is/are the unbiased estimator(s) of  $\theta$  ?

- 1.  $X_{(n)}$
- 2.  $2X_{\left(\frac{n+1}{2}\right)}$

Select the correct answer using the code given below :

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

25. If  $X_1, X_2, X_3, \dots, X_n$  be independent random variables distributed as  $b(1, p)$ , then which of the following statements are correct ?

1.  $T = \sum_{i=1}^n X_i$  is sufficient for  $p$

2.  $T = \sum_{i=1}^n X_i$  is complete for  $p$

3.  $T^2$  is unbiased for  $p^2$

Select the correct answer using the code given below :

- (a) 1 and 2 only
- (b) 2 and 3 only
- (c) 1 and 3 only
- (d) 1, 2 and 3

26. For testing the hypothesis  $H_0 : \theta = \theta_0$  against the alternative  $H_1 : \theta = \theta_1$ ,  $\lambda$  is the likelihood ratio;  $A$  and  $B$  ( $B < A$ ) are the constants. Under the SPRT criteria, consider the following statements :

- 1. If  $\lambda \geq A$ , we terminate the process with the acceptance of  $H_0$
- 2. If  $\lambda \leq B$ , we terminate the process with the rejection of  $H_0$
- 3. If  $B < \lambda < A$ , we continue sampling by taking an additional observation

Which of the above statements is/are correct ?

- (a) 1, 2 and 3
- (b) 1 and 3 only
- (c) 2 and 3 only
- (d) 3 only

27. If  $X_1, X_2, X_3, \dots, X_n$  is a random sample of size  $n$  from Cauchy distribution with pdf

$$f(x) = \frac{1}{\pi[1 + (x - \mu)^2]}; -\infty < x < \infty, \text{ then}$$

which one of the following is true for estimating  $\mu$  ?

- (a) Sample mean is a consistent estimator but not sample median
- (b) Sample median is a consistent estimator but not sample mean
- (c) Both sample mean and sample median are consistent estimators
- (d) Neither sample mean nor sample median is a consistent estimator

28. If  $T_1$  and  $T_2$  are independent and unbiased estimators of parameter  $\theta$  with  $V(T_1) = \sigma_1^2$  and  $V(T_2) = 2\sigma_1^2$ , then the unbiased estimator of  $\theta$  given by  $T = \lambda T_1 + (1 - \lambda)T_2$  has minimum variance if  $\lambda$  equals :

- (a) 2/3
- (b) 1/2
- (c) 1/3
- (d) 1/4

29. Consider the following statements pertaining to likelihood ratio tests :

- 1. The likelihood ratio criterion offers a method of constructing tests of hypotheses which do not fall within the purview of Neyman-Pearson Lemma
- 2. The likelihood ratio test statistic  $\lambda$  is the ratio of the supremum of the likelihood under  $H_0$  to the supremum of the likelihood in the unrestricted parametric space
- 3. The critical region of the likelihood ratio test is the left hand tail of the distribution of  $\lambda$

Which of the above statements are correct ?

- (a) 1 and 2 only
- (b) 2 and 3 only
- (c) 1 and 3 only
- (d) 1, 2 and 3



30. Let  $X_1, X_2, X_3, \dots, X_n$  be a random variable with pdf

$$f(x; \alpha, \beta) = \begin{cases} \frac{1}{\beta^\alpha} e^{-\frac{x}{\beta}} x^{\alpha-1}; & x, \alpha, \beta > 0 \\ 0, & \text{otherwise} \end{cases}$$

What is the consistent estimator for  $\beta^2$  ?

(a)  $\frac{\sum_{i=1}^n x_i^2}{n\alpha(\alpha+1)^2}$

(b)  $\frac{\sum_{i=1}^n x_i^2}{n\alpha^2(\alpha+1)}$

(c)  $\frac{\sum_{i=1}^n x_i^2}{n\alpha(\alpha+1)}$

(d)  $\frac{\sum_{i=1}^n x_i^2}{n^2\alpha(\alpha+1)}$

31. Which one of the following sampling designs is used in conducting the Consumer Expenditure Survey by the National Sample Survey Office (NSSO) ?

- Simple Random Sampling
- Quota Sampling
- Stratified Multi-stage Random Sampling
- Judgement Sampling

32. Given a random sample from

$$f(x, \alpha) = \frac{2}{\alpha^2} (\alpha - x), \quad 0 < x < \alpha.$$

What is the MLE for  $\alpha$  ?

- $x$
- $2x$
- $\frac{x}{2}$
- None of the above

33. Consider the following statements :

- Size of Type-I error is  $P \{ \text{rejecting } H_0 \mid H_0 \text{ is true} \}$
  - Power of a test is  $P \{ \text{rejecting } H_0 \mid H_1 \text{ is true} \}$
  - $H_1 : \mu \neq \mu_0$  is a right tailed hypothesis
- Which of the statements given above are correct ?

- 1 and 2 only
- 2 and 3 only
- 1 and 3 only
- 1, 2 and 3

34. The Manager of a cyber café says that the number of customers visiting on the weekdays followed a Binomial distribution. Which one of the following techniques can be used to test the hypothesis at a given level of significance ?

- Test of significance of mean
- Chi-square test as a test of goodness of fit
- Test of significance of difference between two means
- Correlation analysis



35. What is an unbiased estimator of  $\theta$  for the distribution,  $f(x, \theta) = \theta e^{-\theta x}$ ,  $x \geq 0$  ?

- (a)  $\frac{(n-1)\bar{x}}{n}$
- (b)  $\frac{(n-1)}{n\bar{x}}$
- (c)  $\frac{\bar{x}}{n-1}$
- (d)  $\frac{1}{(n-1)\bar{x}}$

36. Consider the following statements regarding the properties of an estimator :

1. Every consistent estimator is unbiased.
2. If an estimator  $t_n$  based on  $n$  observations is a consistent estimator of  $\theta$ , then for any  $\epsilon > 0$ ,  $\text{Prob} \{|t_n - \theta| < \epsilon\}$  tends to one as  $n \rightarrow \infty$
3. Since consistency is a limiting property, no consistent estimator actually exists as sample size is finite.

Which of the above statements are *not* correct ?

- (a) 2 and 3 only
- (b) 1 and 3 only
- (c) 1 and 2 only
- (d) 1, 2 and 3

37. Consider a random sample  $X_1, X_2, X_3, \dots, X_n$  of size  $n$  from  $N(\theta, \theta^2)$ , the statistic

$$T = \left( \sum_{i=1}^n X_i, \sum_{i=1}^n X_i^2 \right) \text{ for } \theta \text{ is :}$$

- (a) Sufficient and complete
- (b) Sufficient but not complete
- (c) Not sufficient but complete
- (d) Neither sufficient nor complete

38. In usual notations, if a quantity  $\phi(T, \theta)$  exists, whose distribution is independent of  $\theta$ , then which of the following are correct ?

1.  $\phi(T, \theta)$  is called a confidence interval.
2.  $\phi(T, \theta)$  is a pivotal quantity.
3.  $\phi(T, \theta)$  can be used to define  $(1 - \alpha)$  100% confidence interval for  $\theta$ .

Select the correct answer using the code given below :

- (a) 1 and 3 only
- (b) 1 and 2 only
- (c) 2 and 3 only
- (d) 1, 2 and 3

39. Consider the following statements with respect to Neyman-Pearson Lemma if C is a critical region of size  $\alpha$  and k is a constant.

1.  $\frac{L_0}{L_1} \leq k$  inside C

2.  $\frac{L_0}{L_1} \geq k$  outside C

3. C is a most powerful critical region of size  $\alpha$  for testing  $\theta = \theta_0$  against  $\theta = \theta_1$

Which of the above are correct ?

- (a) 1, 2 and 3
- (b) 2 and 3 only
- (c) 1 and 3 only
- (d) 3 only

40. To compare the quality of LED TVs of Brand 'A' and Brand 'B', a statistician collects the following data :

$X_1, X_2, X_3, \dots, X_n$  are the number of dead pixels in a random sample of n Brand 'A' TVs and  $Y_1, Y_2, Y_3, \dots, Y_m$  are the number of dead pixels in a random sample of m Brand 'B' TVs.

Assuming that the  $X_i$ s and  $Y_j$ s are independent Poisson random variables with mean  $\lambda_1$  and  $\lambda_2$  for Brand 'A' and Brand 'B' TVs respectively, the statistician decides to test the hypothesis  $H_0 : \lambda_1 = \lambda_2$  versus  $H_1 : \lambda_1 \neq \lambda_2$ .

Here  $H_0$  and  $H_1$  are, respectively :

- (a) Simple and simple
- (b) Simple and composite
- (c) Composite and simple
- (d) Composite and composite

41. If  $X_i$  follows  $U(\theta, \theta)$  be a random sample for  $i = 1, 2, 3, \dots, n$ , then what is an unbiased estimator of  $\theta$  ?

- (a)  $\bar{X}$
- (b)  $\frac{\bar{X}}{2}$
- (c)  $2\bar{X}$
- (d)  $\sqrt{\bar{X}}$

42. X has a Poisson  $\lambda$  distribution. Then  $(\bar{X} - S^2)$ , where  $\bar{X}$  is the sample mean based on a random sample of size n and

$$S^2 = \frac{\sum (X_i - \bar{X})^2}{n},$$

is an unbiased estimator of :

- (a) 0
- (b)  $\frac{\lambda}{n}$
- (c)  $\lambda - \lambda^2$
- (d)  $\left(1 - \frac{1}{n}\right)\lambda$

43. An unbiased estimator of  $\frac{1}{\theta}$  for the distribution  $f(x, \theta) = \theta(1 - \theta)^{x-1}$ ,  $x = 1, 2, 3, \dots, \infty; 0 < \theta < 1$  is :

- (a)  $\frac{1}{x}$
- (b)  $\frac{1}{n\bar{x}}$
- (c)  $n\bar{x}$
- (d)  $\bar{x}$

44. If  $X_1, X_2, X_3, \dots, X_n$  is a random sample from  $b(1, p)$ , then :

- (a) Sample mean  $\bar{X}$  is unbiased for  $p$  but not consistent
- (b) Sample mean  $\bar{X}$  is consistent for  $p$  but not unbiased
- (c) Sample mean  $\bar{X}$  is unbiased and consistent for  $p$
- (d) Sample mean  $\bar{X}$  is neither unbiased nor consistent for  $p$

45. Consider the following statements :

1. If  $T$  is an unbiased estimator of  $\theta$ , then  $T^2$  is an unbiased estimator of  $\theta^2$
2. If  $T$  is a consistent estimator of  $\theta$ , then  $T^3$  is a consistent estimator of  $\theta^3$

Which of the above statements is/are correct ?

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

46. What is the MLE of  $\theta$  based on a random sample of size  $n$  drawn from a population

with pdf as  $f(x, \theta) = \begin{cases} \theta e^{-\theta x}, & \theta < x < \infty \\ \theta & , \text{ otherwise} \end{cases}$  ?

- (a)  $\bar{x}$
- (b)  $\frac{1}{\bar{x}}$
- (c) Largest observation
- (d) Smallest observation

47. If population variance of an infinite population is  $\sigma^2$  and a sample of 25 items is selected from this population, then what is the standard error of sample mean ?

- (a)  $\frac{\sigma^2}{25}$
- (b)  $\frac{\sigma}{5}$
- (c)  $\frac{\sigma}{25}$
- (d)  $\sigma$

48. Consider the following statements :

Statement-I : An SPRT always provides a decision.

Statement-II : Probability of terminating an SPRT is one.

Which one of the following is correct in respect of the above two statements ?

- (a) Both Statement-I and Statement-II are true and Statement-II is the correct explanation for Statement-I
- (b) Both Statement-I and Statement-II are true but Statement-II is not the correct explanation for Statement-I
- (c) Statement-I is true but Statement-II is false
- (d) Statement-I is false but Statement-II is true

49. Consider the following statements :
1. The purpose of hypothesis testing is to make a judgement about the difference between the sample statistic and the hypothesized population parameter.
  2. When the Null hypothesis is rejected the alternate hypothesis is accepted.
  3. The purpose of hypothesis testing is to test the correctness of the computed value of the sample statistic.
  4. If the Null hypothesis is not rejected, this does not ensure that the Null hypothesis is true.

Which of the above statements are correct ?

- (a) 1, 2 and 3
  - (b) 1, 3 and 4
  - (c) 1, 2 and 4
  - (d) 2, 3 and 4
50. Which one of the following statements is correct?
- (a) The mean of the sampling distribution of mean is greater than the population mean.
  - (b) The sampling distribution of mean approaches normality as the sample size decreases.
  - (c) As the standard error of mean increases, the value of any sample mean will be closer to the value of the population mean.
  - (d) As the standard error of mean decreases, the precision of the sample mean as an estimator of the population mean increases.

51. Suppose  $X_1, X_2, X_3, \dots, X_{10}, Y_1, Y_2, Y_3, \dots, Y_{10}$  are independent random variables where  $X_i$ s have density  $f_X(x) = \frac{1}{\lambda} e^{-\frac{x}{\lambda}}, x > 0$  and  $Y_i$ s have density  $f_Y(y) = \frac{1}{2\lambda} e^{-\frac{y}{2\lambda}}, y > 0$ .

What is MLE of  $\lambda$  based on all  $X_i$ s and  $Y_i$ s ?

- (a)  $\frac{\bar{X} + \bar{Y}}{2}$
  - (b)  $\frac{\bar{X} + \bar{Y}}{3}$
  - (c)  $\frac{2\bar{X} + \bar{Y}}{4}$
  - (d)  $\frac{\bar{X} + 2\bar{Y}}{5}$
52. In a normal population  $N(\mu, \sigma^2)$  with  $\sigma^2 = 4$ , in order to test the null hypothesis  $\mu = \mu_0$  against the alternative  $\mu = \mu_1$ , where  $\mu_1 > \mu_0$  based on a random sample of size  $n$ , the value of  $k$  such that  $\bar{x} > k$  provides a critical region of size  $\alpha = 0.05$  is :

- (a)  $\mu_0 + \frac{1.645}{\sqrt{n}}$
- (b)  $\mu_0 + \frac{3.290}{\sqrt{n}}$
- (c)  $\mu_0 + \frac{1.96}{\sqrt{n}}$
- (d)  $\mu_0 + \frac{3.92}{\sqrt{n}}$



53. For testing the null hypothesis  $H_0 : \mu = \mu_0$  against the alternative  $H_1 : \mu \neq \mu_0$ , at  $\alpha\%$  level of significance on the basis of a random sample of size  $n$  from a normal population with known variance  $\sigma^2 = 1$ , consider the following statements with respect to likelihood ratio criteria ( $\lambda$ ):

1. The value of  $\lambda$  is  $e^{-\frac{n}{2}(\bar{x} - \mu_0)^2}$
2. The null hypothesis is rejected if  $\lambda \leq k, 0 \leq k \leq 1$
3.  $\lambda$  is a ratio of the maximum likelihood function for all values of  $\mu$  in parameter space and maximum likelihood function for all values of  $\mu$  under  $H_0$

Which of the above statements are correct ?

- (a) 1 and 2 only
- (b) 2 and 3 only
- (c) 1 and 3 only
- (d) 1, 2 and 3

54. Consider the following statements :

1. If  $X_1, X_2, X_3, \dots, X_n$  is a random sample from normal population, then the sample variance  $S^2$  is consistent estimator of  $\sigma^2$ .
2. If  $X_1, X_2, X_3, \dots, X_n$  is a random sample from normal population with mean 0, then  $\frac{\sum X_i^2}{n}$  is unbiased estimator of  $\sigma^2$ .

3. If  $X_1, X_2, X_3, \dots, X_n$  is a random sample from an exponential population with probability density function

$$f(x, \theta) = \begin{cases} \frac{1}{\theta} e^{-\frac{x}{\theta}}; & x > 0 \\ 0; & \text{otherwise} \end{cases}, \text{ then } \bar{X} \text{ is}$$

consistent estimator for parameter  $\theta$ .

4. If  $X_1, X_2, X_3, \dots, X_n$  is a random sample from  $U(0, \beta)$ , then the  $n^{\text{th}}$  order statistic  $Y_n$  is unbiased estimator of  $\beta$ .

Which of the above statements are correct ?

- (a) 2, 3 and 4
- (b) 1, 2 and 4
- (c) 1, 3 and 4
- (d) 1, 2 and 3

55. Cramer Rao lower bound of variance for the parameter  $\theta$  of the distribution with pdf

$$f(x, \theta) = \frac{1}{\pi} \frac{1}{1 + (x - \theta)^2} \text{ where } -\infty < x < \infty$$

is :

- (a)  $\frac{1}{n}$
- (b)  $\frac{2}{n}$
- (c)  $\frac{1}{n^2}$
- (d)  $\frac{2}{n^2}$

56. An estimator with large variance is preferred in which one of the following cases ?

- (a)  $X$  follows gamma  $(1, \beta)$
- (b)  $X$  follows  $U(0, \theta)$
- (c)  $X$  follows gamma  $\left(1, \frac{1}{\beta}\right)$
- (d)  $X$  follows gamma  $\left(0, \frac{1}{\theta}\right)$

57. Let  $X_1, X_2, X_3, \dots, X_n$  be independently and normally distributed with mean 0 and variance  $\sigma^2$ . What is the minimum variance unbiased estimator of  $\sigma^2$  ?

- (a)  $\frac{1}{n-1} \sum_{i=1}^n X_i^2$
- (b)  $\frac{1}{n} \sum_{i=1}^n X_i^2$
- (c)  $\frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})^2$
- (d)  $\frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2$

58. If  $X_1, X_2$  and  $X_3$  is a random sample of size 3 from a population with mean  $\mu$  and variance  $\sigma^2$ . What is the value of  $\lambda$  for which

$T_3 = \frac{1}{3}(\lambda X_1 + X_2 + X_3)$  is an unbiased

estimator for  $\mu$  ?

- (a) 1/4
- (b) 1/3
- (c) 1/2
- (d) 1

59. If  $\bar{X}$  and  $S^2$  are the mean and variance of a random sample of size  $n$  from a normal population with mean  $\mu$  and standard deviation  $\sigma$ , then consider the following statements :

1.  $\bar{X}$  is unbiased and consistent estimator of  $\mu$ .
2.  $S^2$  is consistent estimator of  $\sigma^2$ .
3. The random variable  $\frac{(n-1)S^2}{\sigma^2}$  has a Chi-square distribution with  $(n - 1)$  degrees of freedom.

Which of the above statements are correct ?

- (a) 1 and 2 only
- (b) 2 and 3 only
- (c) 1 and 3 only
- (d) 1, 2 and 3

60. If the pdf of a random variable  $X$  is

$$f(x; \theta) = 1 - \theta^2, \quad 0 \leq x \leq \frac{1}{1 - \theta^2} \text{ and if we}$$

reject  $H_0 : \theta = \frac{1}{2}$  against  $H_1 : \theta = \frac{3}{4}$

whenever  $x \geq 1$ , then what is the power of the test ?

- (a) 15/16
- (b) 1/2
- (c) 7/16
- (d) 9/16

61. Consider the following statements :

Statement-I : Fisher's Index number is known as ideal Index number.

Statement-II : Fisher's Index number is the arithmetic mean of Laspeyre's Index number and Paasche's Index number which show upward bias and downward bias respectively.

Which one of the following is correct in respect of the above two statements ?

- (a) Both Statement-I and Statement-II are true and Statement-II is the correct explanation for Statement-I
- (b) Both Statement-I and Statement-II are true but Statement-II is not the correct explanation for Statement-I
- (c) Statement-I is true but Statement-II is false
- (d) Statement-I is false but Statement-II is true

62. If  $X_1, X_2, X_3, \dots, X_n$  is a random sample from  $N(0, \sigma^2)$ , then which one of the following statements is correct ?

(a)  $\sum_{i=1}^n X_i^2$  is minimal sufficient statistic for  $\sigma^2$

(b)  $\sum_{i=1}^n X_i$  is minimal sufficient statistic for  $\sigma^2$

(c)  $\bar{X}$  is minimal sufficient statistic for  $\sigma^2$

(d) There is no minimal sufficient statistic for  $\sigma^2$

63. If  $X_1, X_2, X_3, \dots, X_n$  is a random sample from the uniform distribution on  $[0, a]$  where  $a \in (0, \infty)$ , then which of the following statements are correct ?

1. Maximum likelihood estimator of 'a' is same as the moment estimator of 'a'

2. Maximum likelihood estimator of 'a' is  $X_{(n)} = \text{Max}\{X_1, X_2, X_3, \dots, X_n\} = \hat{a}$

3.  $\frac{n+1}{n} \hat{a}$  is an unbiased estimator of 'a' where  $\hat{a}$  is maximum likelihood estimator of 'a'

4. Method of moments estimator is unbiased estimator of 'a'

Select the correct answer using the code given below :

(a) 2, 3 and 4

(b) 1, 3 and 4

(c) 1, 2 and 4

(d) 1, 2 and 3

64. Randomised test is used for testing parametric hypothesis regarding distribution of a random variable X :

(a) Whenever X has a discrete distribution

(b) Whenever exact size  $\alpha$  test is required

(c) When X is a discrete random variable and exact size  $\alpha$  test is required

(d) When X is a continuous random variable and exact size  $\alpha$  test is required

65. If  $X_1, X_2, X_3, \dots, X_n$  is a random sample from Poisson distribution with parameter  $\lambda$ , then the maximum likelihood estimator of  $\log \lambda$  is given by :

(a)  $\frac{\sum_{i=1}^n X_i}{n}$

(b)  $\log \left( \frac{\sum_{i=1}^n X_i}{n} \right)$

(c)  $\frac{\sum_{i=1}^n X_i}{e^{\sum_{i=1}^n X_i}}$

(d)  $\log \left( \sum_{i=1}^n X_i \right)$

66. Agriculture Census in India is conducted at an interval of :

(a) 3 years

(b) 5 years

(c) 8 years

(d) 10 years

67. Which one of the following sampling designs is used for General Crop Estimation Survey (GCES) ?

- (a) Simple Random Sampling (SRS)
- (b) Judgement Sampling
- (c) Stratified Multi-stage Random Sampling
- (d) Quota Sampling

68. Which of the following are the components of the scheme 'Improvement of Agricultural Statistics' ?

- 1. Timely Reporting Scheme (TRS)
- 2. Improvement of Crop Statistics (ICS)
- 3. Establishment of Agency Reporting Agricultural Statistics (EARAS)
- 4. Studying cost of cultivation of principal crops in India

Select the correct answer using the code given below :

- (a) 2, 3 and 4
- (b) 1, 3 and 4
- (c) 1, 2 and 4
- (d) 1, 2 and 3

69. Minimum Support Prices (MSP) of major agricultural products are recommended to the Government by :

- (a) Directorate of Economics and Statistics
- (b) Directorate of Marketing and Inspection
- (c) Commission for Agricultural Costs and Prices
- (d) Directorate of Rice Development

70. Which one of the following index numbers is used for calculating the Dearness Allowance (DA) of Central Government Employees ?

- 1. Consumer Price Index (Rural, Urban, Combined)
- 2. Consumer Price Index (Industrial Workers)
- 3. Wholesale Price Index
- 4. Index of Industrial Production

Select the correct answer using the code given below :

- (a) 2 only
- (b) 1 and 2 only
- (c) 1, 2 and 3
- (d) 1, 3 and 4

71. Consider a single observation from the population  $f(x, \theta) = \theta e^{-\theta x}$ ,  $0 < x < \infty$ .

Let  $x \geq 1$  be the critical region for testing  $H_0 : \theta = 2$  against  $H_1 : \theta = 1$ . What are the values of type I and type II errors respectively ?

- (a)  $\frac{e-1}{e}, \frac{1}{e^2}$
- (b)  $\frac{1}{e}, \frac{e-1}{e^2}$
- (c)  $\frac{e-1}{e^2}, \frac{1}{e}$
- (d)  $\frac{1}{e^2}, \frac{e-1}{e}$



72. Which one of the following organizations of the Government is responsible for collection, compilation and dissemination of Trade Statistics in India ?

- (a) Central Statistical Office
- (b) Directorate General of Commercial Intelligence and Statistics
- (c) Department of Industrial Policy and Promotion
- (d) Tariff Commission

73. What are the major sources of data on Official Statistics ?

- 1. Census and Sample Surveys
- 2. Administrative Records

Select the correct answer using the code given below :

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

74. Which of the following are the major sources on 'Health Statistics' in India ?

- 1. Sample Registration System (SRS)
- 2. Civil Registration System (CRS)
- 3. National Family Health Survey (NFHS)

Select the correct answer using the code given below :

- (a) 1 and 2 only
- (b) 2 and 3 only
- (c) 1 and 3 only
- (d) 1, 2 and 3

75. Which of the following is/are correct for Consumer Price Index (Rural, Urban, Combined) ?

- 1. Base year is 2012
- 2. Weights have been derived from the Consumer Expenditure Survey conducted by NSSO during 2011-2012

Select the correct answer using the code given below :

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

76. Which of the following are part of mandate of SDRD ?

- 1. Planning of Survey
- 2. Formulation of Sample Design
- 3. Conducting of Sample Survey
- 4. Preparation of Survey Reports

Select the correct answer using the code given below :

- (a) 1, 2 and 4
- (b) 1, 2 and 3
- (c) 1, 3 and 4
- (d) 2, 3 and 4

77. Which of the following equations are correct in respect of National Accounts ?

1.  $NDP = GDP + CFC$
2.  $GNI = GDP + \text{net factor income from abroad}$
3.  $NNI = GNI - CFC$

Select the correct answer using the code given below :

- (a) 1 and 2 only
- (b) 2 and 3 only
- (c) 1 and 3 only
- (d) 1, 2 and 3

78. The first phase of Population Census is associated with which of the following ?

1. House listing
2. Housing Census
3. Population Enumeration

Select the correct answer using the code given below :

- (a) 1 and 2 only
- (b) 2 and 3 only
- (c) 1 and 3 only
- (d) 1, 2 and 3

79. Which of the following is/are correctly matched ?

1. Population Census - Office of Registrar General and Census Commissioner
2. Economic Census - Central Statistics Office

Select the correct answer using the code given below :

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

80. The Unified District Information System for Education (U-DISE), which is an important source of school education data, has been developed by which one of the following ?

- (a) National Council of Educational Research and Training
- (b) National University of Educational Planning and Administration
- (c) University Grants Commission
- (d) National Sample Survey Office

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